## P-wave $\Lambda N$ - $\Sigma N$ coupling and the spin-orbit splitting of ${}^9_{\Lambda}$ Be

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We reexamine the spin-orbit splitting of  $^9_\Lambda \mathrm{Be}$  excited states in terms of the  $SU_6$  quark-model baryon-baryon interaction. The previous folding procedure to generate the  $\Lambda\alpha$  spin-orbit potential from the quark-model  $\Lambda N$  LS interaction kernel predicted three to five times larger values for  $\Delta E_{\ell s} = E_x(3/2^+) - E_x(5/2^+)$  in the model FSS and fss2. This time, we calculate  $\Lambda\alpha$  LS Born kernel, starting from the LS components of the nuclear-matter G-matrix for the  $\Lambda$  hyperon. This framework makes it possible to take full account of an important P-wave  $\Lambda N$ - $\Sigma N$  coupling through the antisymmetric  $LS^{(-)}$  force involved in the Fermi-Breit interaction. We find that the experimental value,  $\Delta E_{\ell s}^{\mathrm{exp}} = 43 \pm 5$  keV, is reproduced by the quark-model G-matrix LS interaction with a Fermi-momentum around  $k_F = 1.0$  fm<sup>-1</sup>, when the model FSS is used in the energy-independent renormalized RGM formalism. On the other hand, the model fss2 gives too large splitting of almost 200 keV. Based on these results and the analysis of the Scheerbaum factors, it is concluded that the model fss2 should be improved to reproduce small single-particle spin-orbit potentials of the  $\Lambda$  hyperon.

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Study of hypernuclei based on the fundamental baryon-baryon interactions is important, since the available scattering data for the hyperon-nucleon (YN)interaction are very scarce. The  $SU_6$  quark-model (QM) baryon-baryon interaction developed by the Kyoto-Niigata group is a comprehensive model for all the octet-baryons  $(B_8)$ , which is formulated in the (3q)-(3q) resonating-group method (RGM) using the spinflavor  $SU_6$  QM wave functions, a colored version of the one-gluon exchange Fermi-Breit interaction, and effective meson-exchange potentials acting between quarks [1]. The early version, the model FSS [2] includes only the scalar (S) and pseudoscalar (PS) meson exchange potentials as the effective meson-exchange potentials, while the renovated one fss2 [3, 4] introduces also the vector (V) meson exchange potentials and the momentumdependent Bryan-Scott terms for the S and V mesons.

As an important application of our QM baryon-baryon interactions, we have carried out Faddeev calculations for the triton and the hypertriton in Ref. [5], in the most reliable framework of using the energy-independent renormalized RGM kernels. The triton binding energy predicted by fss2 is very close to the experimental value with about 350 keV less bound, and the  $\Lambda$  separation energy of the hypertriton is 262 keV vs. the experimental value  $130\pm50$  keV. In the hypertriton calculation, the detailed information is obtained for the central force of the  $\Lambda N$  interaction, since this system is S-wave dominant. On the other hand, the information on the  $\Lambda N$  LS force is obtained, for example, from the very small spin-orbit

 $(\ell s)$  splitting of the 5/2<sup>+</sup> and 3/2<sup>+</sup> excited states of  $^9_\Lambda$ Be,  $\Delta E^{\rm exp}_{\ell s}=43\pm 5$  keV [6]. In the previous papers [7, 8], we performed Faddeev calculations of the two-alpha plus  $\Lambda$  ( $\Lambda\alpha\alpha$ ) system by assuming a simple  $(0s)^4$  shell-model wave function for the  $\alpha$  clusters. For the  $\alpha\alpha$  interaction, a microscopic  $\alpha\alpha$  RGM kernel is used with an effective NN force, Minnesota three-range force. The  $\Lambda\alpha$  interaction is generated from a simple two-range Gaussian central potential (SB potential), which simulates the S-wave phase-shifts of the  $\Lambda N$  interaction of fss2 with a slight modification to fit the  $\Lambda\alpha$  bound state. The Pauli forbidden states between the two  $\alpha$  clusters are exactly eliminated in the three-cluster Faddeev formalism using two-cluster RGM kernels [9, 10].

The origin of the  $5/2^+$  and  $3/2^+$  splitting in the  $\Lambda\alpha\alpha$ cluster model is the spin-orbit potential between  $\Lambda$  and one of the  $\alpha$  clusters, which is known to be very small due to the strong cancellation between the symmetric (LS)and antisymmetric  $(LS^{(-)})$  LS forces of the  $\Lambda N$  interaction. As a first step, we directly used in Ref. [8] the QM  $\Lambda N \ LS \ RGM \ kernel to generate the <math>\Lambda \alpha \ LS$  potential by simple  $\alpha$ -cluster folding. In this procedure, the QM  $\Lambda N LS$  interaction of FSS or fss2 predicted 3 to 5 times larger values for  $\Delta E_{\ell s}$ , which is not much improved in comparison with the results of Nijmegen simulated potentials [11]. It was pointed out in Ref. [8] that a further reduction is possible in the model FSS, if one can properly take into account the short-range correlation of the P-wave  $\Lambda N - \Sigma N$  coupling by the  $LS^{(-)}$  force. This was conjectured through the analysis of the Scheerbaum factors for the single-particle (s.p.) spin-orbit potentials, calculated in the G-matrix formalism.

In this new calculation, we generate  $\Lambda \alpha \ LS$  Born kernel from the LS component of the nuclear-matter G-matrix

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for the  $\Lambda$  hyperon. For the  $(0s)^4$   $\alpha$ -cluster folding, a new method developed in Ref. [12] is employed to derive direct and knock-on terms of the interaction Born kernel from the hyperon-nucleon G-matrices with explicit treatments of the nonlocality and the center-of-mass (c.m.) motion between the hyperon and the  $\alpha$  cluster. The G-matrix calculations are carried out by assuming a constant Fermi momentum  $k_F$ , since the local density approximation does not seem to work in light nuclear systems. The G-matrix equation is solved for the energy-independent QM baryon-baryon interaction formulated in the renormalized RGM [13, 14], and the continuous prescription for intermediate spectra is employed. A similar procedure of the energy-independent renormalized RGM is also used for the  $\alpha\alpha$  RGM kernel.

We start from the  $\Lambda N - \Sigma N$  coupled-channel G-matrix equation [15, 16]

$$G_{\gamma lpha}(oldsymbol{p},oldsymbol{q};K,\omega,k_F) = V_{\gamma lpha}^{
m RGM}(oldsymbol{p},oldsymbol{q}) + \sum_eta rac{1}{(2\pi)^3} \int d\,oldsymbol{k}$$

$$\times V_{\gamma\beta}^{\text{RGM}}(\boldsymbol{p}, \boldsymbol{k}) \frac{Q_{\beta}(k, K)}{e_{\beta}(k, K; \omega)} G_{\beta\alpha}(\boldsymbol{k}, \boldsymbol{q}; K, \omega, k_F) , \qquad (1)$$

where  $Q_{\beta}(k,K)$  stands for the angle-averaged Pauli operator and  $e_{\beta}(k,K;\omega) = \omega - E_b(k_1) - E_N(k_2)$  is the energy denominator. Here, we use the notation  $\beta$  etc. to specify  $\Lambda N$  or  $\Sigma N$  channel and b for the corresponding  $\Lambda$  or  $\Sigma$ . Explicit expressions for  $Q_{\beta}$  and  $k_i$  are given in Ref. [15]. The s.p. energy  $E_b(k)$  is defined by

$$E_b(k) = M_b + \frac{\hbar^2}{2M_b}k^2 + U_b(k) \quad , \tag{2}$$

with  $U_b(k)$  and  $M_b$  being the s.p. potential and the mass for the baryon b, respectively. The starting energy  $\omega$  is a sum of the s.p. energies of two interacting baryons:

$$\omega = E_a(q_1) + E_N(q_2) = M_a + M_N + \frac{\hbar^2}{2(M_a + M_N)} K^2 + \frac{\hbar^2}{2\mu_0} q^2 + U_a(q_1) + U_N(q_2) , \quad (3)$$

where K and q are the total and relative momenta corresponding to the initial s.p. momenta  $q_1$  and  $q_2$ . The s.p. potentials  $U_{\Lambda}(q_1)$ ,  $U_{\Sigma}(q_1)$  and  $U_{N}(q_2)$  are determined self-consistently in the standard procedure by assuming a constant  $k_F$ . The determination of  $q_1$  and  $q_2$  is discussed below, in relation to the folding formula of the  $\Lambda \alpha$  Born kernel.

We employ the energy-independent Born kernel [13, 14] for the  $\Lambda N$ - $\Sigma N$  coupling:

$$V^{\text{RGM}}(\boldsymbol{p}, \boldsymbol{q}) = V_{\text{D}}(\boldsymbol{p}, \boldsymbol{q}) + G(\boldsymbol{p}, \boldsymbol{q}) + W(\boldsymbol{p}, \boldsymbol{q}) ,$$
 (4)

with

$$W = \frac{1}{\sqrt{N}} (T_r + V_D + G) \frac{1}{\sqrt{N}} - (T_r + V_D + G) . \quad (5)$$

Here,  $T_r$  is the kinetic-energy operator for the relative motion and N stands for the normalization kernel in the

 $\Lambda N - \Sigma N$  space. This energy-independent treatment of the QM baryon-baryon interaction in the G-matrix formalism requires some kind of orthogonalization procedure for the  $\Lambda N - \Sigma N^{-1} S_0$  channel, since this channel involves a Pauli forbidden state at the quark level. The redundant correction of the G-matrix is carried out in a similar way to the RGM T-matrix used in the Faddeev formalism. The details will be published elsewhere [17].

The derived G-matrix interaction is expressed in the form of the invariant G-matrices as [12]

$$G_{\Lambda N,\Lambda N}(\boldsymbol{p}, \boldsymbol{p}'; K, \omega, k_F) = \langle \Lambda N \mid G(\boldsymbol{p}, \boldsymbol{p}'; K, \omega, k_F) - G(\boldsymbol{p}, -\boldsymbol{p}'; K, \omega, k_F) P_{\sigma} P_{\tau} \mid \Lambda N \rangle$$

$$= g_0 + g_{ss}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + h_0 i \widehat{\boldsymbol{n}} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$$

$$+ h_- i \widehat{\boldsymbol{n}} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + \cdots . \tag{6}$$

Here,  $\widehat{\boldsymbol{n}} = \boldsymbol{n}/|\boldsymbol{n}|$  with  $\boldsymbol{n} = [\boldsymbol{p}' \times \boldsymbol{p}]$ , and the invariant functions,  $g_0$  (central),  $g_{ss}$  (spin-spin),  $h_0$  (LS),  $h_-$  ( $LS^{(-)}$ ), etc. depend on  $p = |\boldsymbol{p}|$ ,  $p' = |\boldsymbol{p}'|$ , and  $\cos\theta = (\widehat{\boldsymbol{p}} \cdot \widehat{\boldsymbol{p}}')$ , as well as the G-matrix parameters K,  $\omega$  and  $k_F$ . These are expressed by the partial-wave components of the G-matrix as in Appendix D of Ref. [18]. The spin-spin term and the omitted noncentral terms in Eq. (6) do not contribute to the  $\Lambda\alpha$  Born kernel, due to the spin-saturated property of the  $\alpha$  cluster.

The  $\Lambda \alpha$  Born kernel

$$V_{\Lambda\alpha}(\boldsymbol{q}_f, \boldsymbol{q}_i) = V^C(\boldsymbol{q}_f, \boldsymbol{q}_i) + V^{LS}(\boldsymbol{q}_f, \boldsymbol{q}_i) \ i \hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}_{\Lambda} , \quad (7)$$

is calculated by the folding formula derived in Ref. [12]. Here,  $\hat{\boldsymbol{n}} = \boldsymbol{n}/|\boldsymbol{n}|$  with  $\boldsymbol{n} = [\boldsymbol{q}_i \times \boldsymbol{q}_f]$ . For the angular momentum projection of the central and LS terms, it is convenient to use the momentum transfer  $\boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i$  and the local momentum  $\boldsymbol{q} = (\boldsymbol{q}_f + \boldsymbol{q}_i)/2$  of the  $\Lambda\alpha$  system, together with the similar relationship  $\boldsymbol{k}' = \boldsymbol{p} - \boldsymbol{p}'$  and  $\boldsymbol{q}' = (\boldsymbol{p} + \boldsymbol{p}')/2$  at the two-baryon level. For example, the central Born kernel  $V^C(\boldsymbol{q}_f, \boldsymbol{q}_i)$  in Eq. (7) is expressed as

$$V^{C}(\mathbf{q}_{f}, \mathbf{q}_{i}) = \mathcal{V}^{C}(\mathbf{k}, \mathbf{q}) = 4e^{-\frac{3}{32\nu}k^{2}} \left(\frac{2(1+\xi)^{2}}{3\pi\nu}\right)^{\frac{3}{2}}$$

$$\times \int d\mathbf{q}' \exp\left\{-\frac{2(1+\xi)^{2}}{3\nu} \left(\mathbf{q}' - \frac{1+4\xi}{4(1+\xi)}\mathbf{q}\right)^{2}\right\}$$

$$\times g_{0}(\mathbf{q}' + \mathbf{k}/2, \mathbf{q}' - \mathbf{k}/2; (1+\xi)|\mathbf{q} - \mathbf{q}'|, \omega, k_{F}), \quad (8)$$

where  $\xi = M_N/M_\Lambda$  and  $\nu$  is the harmonic oscillator size parameter of the  $\alpha$  cluster. Transformations from  $g_0$  to  $\mathcal{G}_0(\mathbf{k}',\mathbf{q}') = g_0(\mathbf{q}' + \mathbf{k}'/2,\mathbf{q}' - \mathbf{k}'/2;(1+\xi)|\mathbf{q} - \mathbf{q}'|,\omega,k_F)$  and from  $\mathcal{V}^C(\mathbf{k},\mathbf{q})$  to  $V^C(\mathbf{q}_f,\mathbf{q}_i)$  are carried out numerically for their partial-wave components. For the direct and knock-on terms like Eq. (8), we find  $\mathbf{k} = \mathbf{k}'$ . The relationship  $K = (1+\xi)|\mathbf{q} - \mathbf{q}'|$  in Eq. (8) implies that the local momentum  $\mathbf{q}$  of the  $\Lambda\alpha$  Born kernel corresponds to the initial momentum  $\mathbf{q}_1$  for the G-matrix equation and  $\mathbf{q}'$  to the relative momentum  $\mathbf{q}$  in Eq. (3). The standard angular-averaging procedure for the  $\mathbf{q}'$  integral in Eq. (8) gives the starting energy  $\omega$  as a function of  $q_1$  and q, which we call  $\omega(q_1,q)$  prescription. We therefore

assign  $q_1 \to q = |\mathbf{q}|$  and  $q \to q' = |\mathbf{q}'|$  in Eq. (8) and obtain a simple folding formula for the partial-wave components [12]. In the application to the  $n\alpha$  RGM using the G-matrix NN interaction [19], we have improved this method to make possible the treatment of other interaction types. This improved version specifies  $\omega$  as a function of q and K (namely,  $\omega = \omega(q, K)$ ) by applying the angular-averaging procedure to K. The explicit angular-momentum projection on the  $\mathbf{q}'$ -dependence in K makes it possible to deal with the Pauli-forbidden state in the  $n\alpha$  relative motion. In the following, we will also show the results by this  $\omega(q, K)$  prescription, but the difference from the  $\omega(q_1, q)$  prescription is only quantitative.

For the Faddeev calculation, we use the same conditions as used in Ref. [8], except for the exchange mixture parameter u of the SB  $\Lambda N$  potential. We here use u=1, which is the same value as in Ref. [7]. The increase from u = 0.82 to u = 1 is because the energy-independent treatment of the  $\alpha\alpha$  RGM kernel gives slightly more repulsive effect than the previous energy-dependent treatment. With this u value, the ground-state energy of  ${}^9_{\Lambda}$  Be is  $-6.596 \sim -6.598$  MeV, which corresponds to the experimental value  $-6.62\pm0.04$  MeV. We have not used the central  $\Lambda \alpha$  Born kernel obtained from the G-matrix calculation, since the interaction strength is rather sensitive to the assumed  $k_F$  value. For example, the  $\Lambda \alpha$  boundstate energy, predicted by FSS in the  $\omega(q_1, q)$  (or  $\omega(q, K)$ ) prescription, is -2.95 (-2.46) MeV for  $k_F = 1.20$  fm<sup>-</sup> and -4.04 (-3.43) MeV for  $k_F = 1.07$  fm<sup>-1</sup>, compared with the experimental value  $-3.12 \pm 0.02$  MeV. The purpose of the present investigation is to examine the LScomponent from the QM  $\Lambda N - \Sigma N$  interaction.

Table I shows the results of Faddeev calculations in the jj-coupling scheme, obtained by using the QM Gmatrix  $\Lambda \alpha$  LS Born kernel. The Fermi momenta  $k_F =$ 1.07, 1.20, and 1.35 fm<sup>-1</sup> correspond to the densities  $\rho =$  $0.5 \rho_0$ ,  $0.7 \rho_0$ , and  $\rho_0$ , respectively, with  $\rho_0 = 0.17 \text{ fm}^{-3}$ being the normal saturation density. The final values for the  $\ell s$  splitting of the  $5/2^+$  and  $3/2^+$  excited states are  $\Delta E_{\ell s} = 39$  - 96 keV for FSS and 205 - 223 keV for fss2, depending on the  $k_F$  values in the range of 1.07 -1.35 fm<sup>-1</sup>. If the  $\omega(q,K)$  prescription is used, the results by fss2 are similar, but those by FSS are 56 - 118 keV. A smaller  $k_F$  value gives a smaller  $\ell s$  splitting. If we compare these results with the experimental value  $\Delta E_{\ell s}^{\rm exp} = 43 \pm 5$  keV, we find that the model FSS can reproduce the experimental value if the  $k_F$  value around  $1.09 - 1.02 \text{ fm}^{-1}$  is used. We find that the excitation energies of the  $5/2^+$  and  $3/2^+$  states are almost 120 keV too low, when the  $\ell s$  splitting is correctly reproduced with FSS. This is the result when the energy-independent renormalized RGM kernels are used for the  $\alpha\alpha$  RGM kernel and for the QM baryon-baryon interaction. On the other hand, fss2 gives too large values around 200 keV.

These results are consistent with the tendency of the Scheerbaum factor  $S_{\Lambda}$  in the nuclear matter. Table I also lists the Scheerbaum factor  $S_{\Lambda}$  in symmetric matter, indicating the strength of the spin-orbit potentials of the  $\Lambda$ 

hyperon. A similar quantity can be derived for the zeromomentum Wigner transform calculated from the  $\Lambda \alpha LS$ Born kernel in Eq. (7) (see Eq. (2.47) of Ref. [12]). This quantity, that we call the Scheerbaum-like factor  $\widetilde{S}_{\Lambda}$ , is expected to give a better measure for the strength of the  $\Lambda \alpha$  spin-orbit interaction, since it deals with the recoil effect of the  $\alpha$  cluster due to the correct treatment of the c.m. motion in the  $\alpha$ -cluster folding. The recoil effect is about 20 - 30% and is by no means small, as discussed in our previous paper [8]. We find that the strong cancellation between the LS and  $LS^{(-)}$  forces takes place in the QM Fermi-Breit interaction for the P-wave  $\Lambda N - \Sigma N$ coupling in the  ${}^{1}P_{1}-{}^{3}P_{1}$  state, when the G-matrix equation is solved especially in low-density nuclear matter. This is most prominently exhibited in the model FSS. The spin-orbit contribution from the effective-meson exchange potentials in fss2 does not lead to the small  $\ell s$ splitting of the  $\Lambda$  hyperon, since the scalar-meson exchange LS force contains only the ordinary LS and does not produce the  $LS^{(-)}$  force.

The previous energy-dependent treatment of the RGM kernels yields the results qualitatively similar to the present investigation. The reduction of the energy splitting and the  $S_{\Lambda}$ ,  $\widetilde{S}_{\Lambda}$  factors for the smaller  $k_F$  values is very drastic for FSS. We find that  $k_F = 1.25 \text{ fm}^{-1}$  will give the correct value of  $\Delta E_{\ell s}$  if FSS is used. On the other hand, the model fss2 gives almost no reduction for the smaller  $k_F$  values.

In spite of the successful reproduction of the  ${}^9_{\Lambda}{\rm Be}~\ell s$ splitting by the model FSS, there still remains an important issue on the P-wave characteristics of the  $\Lambda N$ interaction. Owing to the very strong P-wave  $\Lambda N$ - $\Sigma N$ coupling in FSS, the  ${}^{3}P_{1}$   $\Sigma N$  resonance moves to the  ${}^{1}P_{1}$  $\Lambda N$  channel, resulting in a very broad step-like resonance in this channel, as seen in Fig. 14 of Ref. [1]. As the result, the cusp structure in the  $\Lambda p$  total elastic cross sections at the  $\Sigma N$  threshold is largely enhanced compared with that of the fss2 prediction, which is clearly overestimated even from the present experimental data with large error bars. See Fig. 19 (e) of [1]. The original  ${}^{3}P_{1}$   $\Sigma N$  resonance is caused by the attractive Pauli effect from the exchange kinetic-energy kernel, related to the Pauli forbidden (11)<sub>s</sub>  $SU_3$  state for the most compact  $(0s)^6$ six-quark configuration in the flavor-symmetric channel. The resonance behavior in the  $\Lambda N - \Sigma N(I = 1/2)^{-1} P_1$  ${}^3P_1$  state sensitively depends on the strength of the  $LS^{(-)}$ force and the strength of the attractive central force in the  $\Sigma N(I=1/2)$  channel. Furthermore, the central  $\Lambda N$ interaction of FSS has a problem that the  ${}^{1}S_{0}$  interaction is too attractive, in comparison with the  ${}^{3}S_{1}$  interaction. For this reason, the hypertriton calculation in Ref. [5] leads to the large overbinding when FSS is used. These inconsistencies between the central and LS components of the  $\Lambda N$  interaction imply that we still need better models to describe the  $\Lambda$  hypernuclei by means of the  $SU_6$  QM baryon-baryon interaction.

Summarizing this work, we have carried out  $\Lambda \alpha \alpha$  Faddeev calculations by employing the  $\Lambda \alpha$  LS Born kernel

TABLE I: The Scheerbaum factor  $S_{\Lambda}$  for symmetric nuclear matter, the Scheerbaum-like factor  $\widetilde{S}_{\Lambda}$  from the  $\Lambda \alpha$  zero-momentum Wigner transform for the spin-orbit force, and the energy splitting,  $\Delta E_{\ell s} = E_{\rm x}(3/2^+) - E_{\rm x}(5/2^+)$ , of the  ${}^9_{\Lambda}$ Be excited states predicted from the  $\alpha \alpha \Lambda$  Faddeev calculations using the QM G-matrix  $\Lambda \alpha$  LS Born kernel. The model is fss2 and FSS and the continuous prescription is used for intermediate spectra in the G-matrix calculation. The energy-independent renormalized RGM kernel is used for the  $\alpha \alpha$  RGM kernel and for the QM baryon-baryon interactions. The  $\omega(q_1,q)$  prescription is used for the starting energies. The results by the  $\omega(q,K)$  prescription are also shown in the parentheses. (See the text.)

	1. (c1)	1.07	1.00	1.95
	$k_F  (\mathrm{fm}^{-1})$	1.07	1.20	1.35
	$ ho/ ho_0$	0.5	0.7	1
G-matrix	fss2	-11.8	-12.1	-12.3
$S_{\Lambda} ({ m MeVfm^5})$	FSS	-4.1	-5.2	-6.3
$\Lambda \alpha$	fss2	-14.7 (-14.8)	$-15.6 \; (-15.6)$	-16.3 (-16.3)
$\widetilde{S}_{\Lambda} (\mathrm{MeV}\mathrm{fm}^5)$	FSS	-3.5 (-4.6)	-5.7 (-6.7)	-7.6 (-8.7)
$\Lambda \alpha \alpha$ Faddeev	fss2	205 (204)	213 (214)	223 (220)
$\Delta E_{\ell s} \; (\text{keV})$	FSS	39 (56)	68 (87)	96 (118)
$\Delta E_{\ell s}^{ m exp}~({ m keV})$			$43 \pm 5$	

generated from the LS components of the nuclear-matter G-matrix for the  $\Lambda$  hyperon. One of our  $SU_6$  QM baryonbaryon interaction FSS can reproduce the very small  $\ell s$ splitting of  $^9_\Lambda \text{Be}$  excited states,  $\Delta E^{\text{exp}}_{\ell s} = 43 \pm 5$ , when an appropriate  $k_F$  value corresponding to the half density of the normal saturation density is employed in the Gmatrix calculation. The explicit value of  $k_F$  depends on the model construction even within the framework of the  $\Lambda \alpha \alpha$  cluster model for  ${}^{9}_{\Lambda} \text{Be}$ ;  $k_F = 1.09 \text{ fm}^{-1}$  for the model FSS with the  $\omega(q_1,q)$  prescription and 1.02 fm<sup>-1</sup> with the  $\omega(q,K)$  prescription, when the energy-independent renormalized RGM kernels are used for the  $\alpha\alpha$  RGM kernel and for the QM baryon-baryon interaction. The previous energy-dependent version of the RGM kernel requires  $k_F = 1.25 \text{ fm}^{-1}$  to reproduce  $\Delta E_{\ell s}$  by FSS. On the other hand, the model fss2 gives too large splitting of almost 200 keV. An essential ingredient of the present formalism is to take into account an important P-wave  $\Lambda N - \Sigma N$  coupling through the antisymmetric  $LS^{(-)}$  force involved in the Fermi-Breit interaction. From the present results and the analysis of the Scheerbaum factors for the s.p. spin-orbit potentials, we conclude that the spin-orbit

contribution from the effective meson-exchange potentials in fss2 needs to be improved to reproduce the small spin-orbit interaction of the  $\Lambda$  hyperon, experimentally observed. Construction of a new model with consistent  $\Lambda N$  central and LS interactions is now in progress.

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